Detecting a regime change (Doo Re Song)

Definition **Regime change** in time-series analysis is a modeling concept that the properties of the stochastic process dynamically change over the temporal space [4-1].

For instance, suppose an AR(p=1) process $r_t = c_{m_t} + \beta_{m_t} r_{t-1} + z_t \ldots z_t \sim \mathcal{N}(0, \sigma^2_{m_t})$. The label m_t here represents the mode (or *regime*) that the observation r may belong to at time t among possible group of modes M. Depending on m_t, we estimate the parameters - $\theta_{m_t} = \{c_{m_t}, \beta_{m_t}, \sigma_{m_t}\}$ - of the AR model differently for each t, which means the model parametric properties switch time to time. Assume $\dim(M) = 2$ and the mode transition is Markovian with time-invariant probabilities, then the transition matrix (T) among the mode categories becomes

$$P(m_t|m_{t-1}) = \begin{bmatrix} p_{0,0} & p_{0,1} \\ 1 - p_{0,0} & 1 - p_{0,1} \end{bmatrix}$$

Under this **Markov regime switching model** - hierarchical stochastic process that has a latent Markovian (stochastic process) that determines the (hyper)parameters of the downstream observation dynamics -, the joint distribution of the observation and the mode becomes

$$P(r_t = r, m_t = m|r_{t-1}, m_{t-1}, \{\theta_m\}_M, \underline{\underline{T}}) =$$

$$P(r_t = r|r_{t-1}, z_t, \{c_{m_t=m}, \beta_{m_t=m}\})\mathcal{N}(z_t|0, \sigma^2_{m_t=m})P(m_t = m|m_{t-1}, \{p_{0,0}, p_{0,1}\})$$

, by which one can extract the marginal distribution of r_t or m_t conditioned to the last step's. The parameters to calibrate in this model are $\{\theta_m\}_{m \in M}$ to calibrate with observations and initial mode via approximate optimisation algorithm such as Expectation-Maximisation due to the hidden state nature of the mode [4-2]. Detecting the regime change or **change point detection** is to infer the event where the mode changes from the previous one at the latent process, in the above model it is a classification task of estimating $P(m_t = m|r_t, r_{t-1}, m_{t-1}, \{\theta\}_M)$ where the true labels are never known.

Compared to another latent state-space model such as Kalman Filter, this Markov switching model provide categorical representation of the latent space that also governs the group of observation process parameters such that the model offers interpretability on underlying dynamics.

Lastly, the method of dynamic regime is not limited to the markov switching model with the regime being the latent variable to discover post-optimisation. For instance, one can label the regimes directly via conditioned clustering and optimise the regime labeller as well as the downstream observation process stack-by-stack [4-4]. Industrial practice also apply the concept of regime to extract trend with before any stochastic observation process is considered, and the regime model could be a pure deterministic function - such as piece-wise interpolation. However, the markov switching model offers broader probasilitic analysis due to the generative model nature of the markov chain with end-to-end optimisation property.

Description Regime change is a concept of timeseries modeling that underlying variables of the dynamics - such as latent variables or stochastic process paramaeters - could vary in temporal space. The markov regime switching model is the baseline choice in this subject where the regime is categorised as a latent variable `mode` that governs the parameters of the observation stochastic process.

Demonstration In this work, we consider fund rate of the US federal reserve bank [4-5] as the subject to analyse the regime changing property of its dynamics. The FED funds rate is the reserve lending rate supported by the FED for a selected group of depository institutions including commercial/investment banks, credit unions, and etc. The rate is governed by the FED as the channel of its monetary policy implementation under the aim of achieving their monetary ecomonic outlook. The driving factors of their decision include the realised US inflation rate, the US GDP, the US employment rate against their target numbers [4-6], and the weights of each factor vary by time.

In this demonstration, we apply the abovementioned markov switching model to describe the regime changing dynamics of the FED funds rate (`FEDFUNDS`). First, we assess the timeseries property of the rate itself and glimpse on our the regime switching model works by itself while autoregression is considered. Next, the main line of analysis here is to assess how the well-known driving factors contribute to explain the funds rate decision, which are FRED code itself or combination:

- "Consumer Price Index (CPI)" for inflation: `CORESTICKM159SFRBATL`
- "GDP output gap" for GDP divergence between realised and potential:`1 - GDPPOT/GDPC1`
- "Unemployment rate" for employment status: `UNRATE`

, motivated from the Taylor's rule [4-7] with unemployment rate added to describe the modern FED target and all are available in the FED data api.

Concretely, the inference pass of the rate is modelled as an OLS on I(1) with dynamic parameters by mode of the time:

$$\Delta^{(1)} r_t = c_{m_t} + \underline{w_{m_t}} \cdot \underline{x_t} + z_t \ldots z_t \sim \mathcal{N}(0, \sigma^2_{m_t}),\ m_t \sim \underline{\underline{T}} \cdot m^*_{t-1}$$

, where $m^*_{t-1} = argmax_m P(m_{t-1} | \Delta^{(1)} r_{t-1}, \Delta^{(1)} r_{t-2}, m^*_{t-2})$ is the max aposterior mode in the previous timestep and $\underline{x_t}$ is the economic indicators as exogenous variables - the FED receives the data before publication s.t we don't need to apply delay for our model. The backward pass of the markovian model is the likelihood of the rate observations multiplied by the initial mode and parameters prior as

$$P(\{w_m, \theta_m\}_{m \in M}, \underline{\underline{T}}, m_0 | \{\Delta^{(1)} r_t, x_t\})$$

$$\propto \Pi_i P(\Delta^{(1)} r_i, m_i | \underline{x_i}, \Delta^{(1)} r_{i-1}, m_{i-1}, \{w_m, \theta_m\}_{m \in M}, \underline{\underline{T}}) P(m_0) P(\{w_m, \theta_m\}_{m \in M}, \underline{\underline{T}})$$

, for which we may apply the Expectation-Maximisation algorithm to fit parameters over multiple regime mode to optimise with the likelihood [4-2].

To evaluate the effectiveness of this dynamic regime model assumption, we assess the quality of the calibration by analysing the key test-statistics around the model calibration as well as residuals. Finally, we inspect the quality of the regmine mode labelling done by the markov switch model and reveal its characteristics.

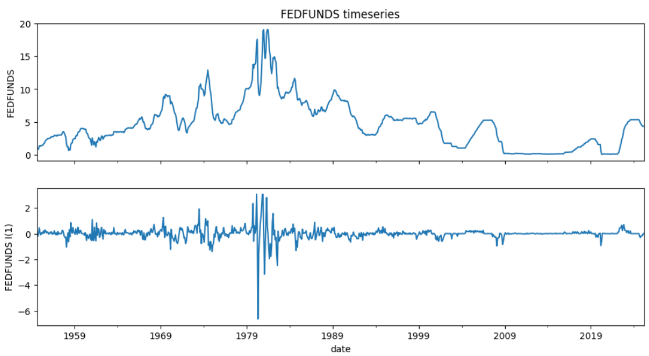
Diagram This demonstration is the extension of [4-8], and the workflow is as follows

1. Assess the fund rate in terms of stationarity
2. Apply the Markov switching regression model and verify its calibration validity with test stats
3. Qualitatively analyse the resultant regime dynamics to assess the nature of each regime and the labelling quality

The markov switch model evaluates each month to align with the frequency of economic indicators data. Each regime is allowed to have their own unique variance as optimisable parameters. The input data to the model is not standardised so please do not put any meaning on the magnitude of the coefficients of the OLS.

Figure 1 contains the key images to express step 1, and Figure 2 explains the rest. Please read each figure from top to bottom, left to right.

Figure 1: FED funds rate stationarity	Chart, unit root tests, and partial autocorrelation

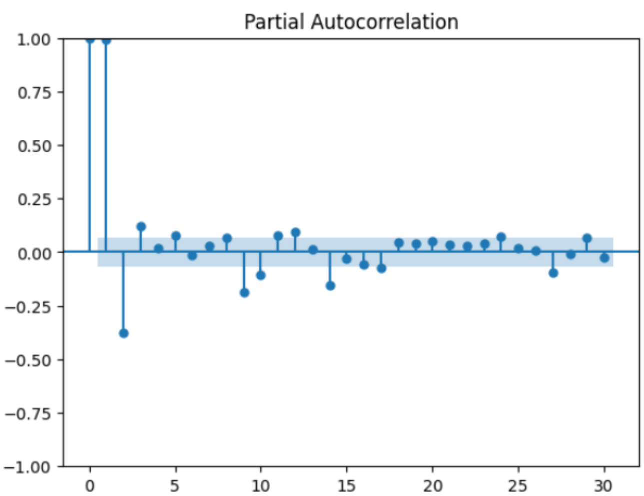



```
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -2.980
P-value                         0.037
Lags                               17
-------------------------------------

Trend: Constant
Critical Values: -3.44 (1%), -2.87 (5%), -2.57 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
```

```
    KPSS Stationarity Test Results
=====================================
Test Statistic                  1.260
P-value                         0.001
Lags                               18
-------------------------------------

Trend: Constant
Critical Values: 0.74 (1%), 0.46 (5%), 0.35 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
```



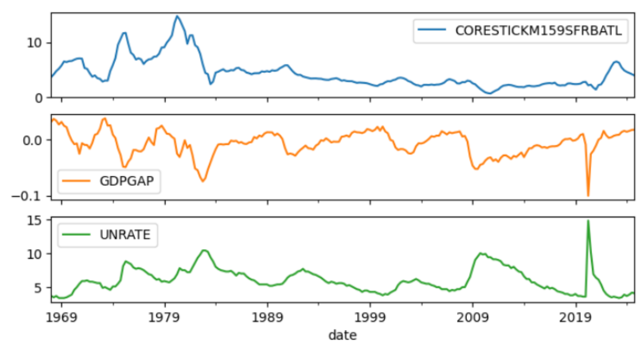
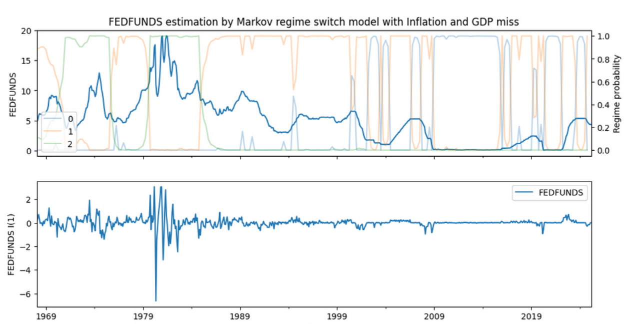
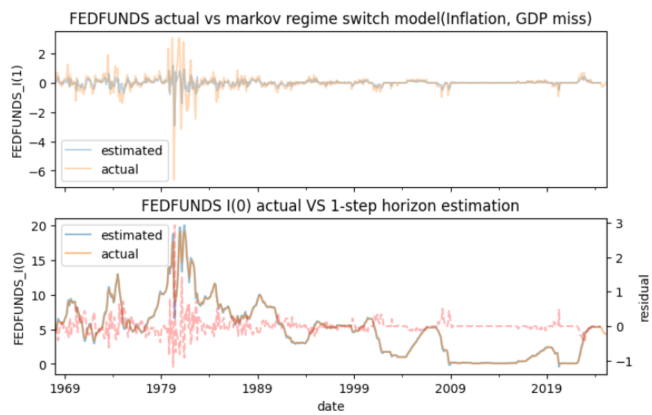


Figure 2: FED funds rate modeling via inflation and GDP gap with regime	Markov switch model calibration and pastcast

```
                        Markov Switching Model Results
==============================================================================
Dep. Variable:                FEDFUNDS   No. Observations:                  228
Model:                MarkovRegression   Log Likelihood                  61.301
Date:                 Sun, 06 Apr 2025   AIC                            -80.602
Time:                         19:51:46   BIC                             -8.586
Sample:                     01-01-1968   HQIC                           -51.546
                          - 10-01-2024
Covariance Type:                approx
                             Regime 0 parameters
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0136      0.019      0.703      0.482      -0.024       0.052
x1            -0.0037      0.004     -0.940      0.347      -0.012       0.004
x2             0.4314      0.220      1.965      0.049       0.001       0.862
x3             0.0002      0.002      0.072      0.943      -0.005       0.005
sigma2         0.0002   4.32e-05      4.771      0.000       0.000       0.000
                             Regime 1 parameters
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0393      0.114      0.344      0.731      -0.184       0.263
x1             0.0138      0.012      1.149      0.250      -0.010       0.037
x2             5.5645      1.915      2.906      0.004       1.811       9.318
x3            -0.0213      0.022     -0.969      0.332      -0.064       0.022
sigma2         0.0474      0.006      7.311      0.000       0.035       0.060
                             Regime 2 parameters
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.9252      0.932     -0.993      0.321      -2.752       0.902
x1             0.0417      0.035      1.206      0.228      -0.026       0.109
x2            18.0406      9.127      1.977      0.048       0.151      35.930
x3             0.1406      0.139      1.013      0.311      -0.132       0.413
sigma2         0.5767      0.135      4.281      0.000       0.313       0.841
                         Regime transition parameters
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
p[0->0]        0.8156      0.000   2116.262      0.000       0.815       0.816
p[1->0]        0.0819      0.028      2.874      0.004       0.026       0.138
p[2->0]     9.137e-12        nan        nan        nan         nan         nan
p[0->1]        0.1844      0.000   1006.352      0.000       0.184       0.185
p[1->1]        0.9008      0.031     29.124      0.000       0.840       0.961
p[2->1]        0.0537      0.038      1.418      0.156      -0.021       0.128
==============================================================================
```




```
                          0          1         2
expected_durations  8.571892  16.137636  8.504109
```




```
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -5.363
P-value                         0.000
Lags                               17
-------------------------------------

Trend: Constant
Critical Values: -3.44 (1%), -2.87 (5%), -2.57 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
```

```
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.109
P-value                         0.545
Lags                                6
-------------------------------------

Trend: Constant
Critical Values: 0.74 (1%), 0.46 (5%), 0.35 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.


Durbin-Watson: 1.5657236469886298
```

Diagnosis Figure 1-1/2/3 indicates that the fed funds rate timeseries is non-stationary with unignorable autocorrelation to previous policy decision. Seeing the economic indicators in Figure 1-4, we can clearly observe that the rate correlates with the CPI in sync with the funds rate in 1970/80s as well as post-Covid, and the GDP offshoot gap slightly *leads* and its development is less spiky (=smooth) than that of the funds rate or the CPI. The unemployment rate almost completely anti-correlates with the GDP gap but as a *lagging* factor except the Covid period. High ACF on early lags with ADF H_{null} acceptance indicated that the funds rate shall better be modelled with I(1) process.

Figure 2-2 shows the calibration result in chart format. Whilst the model catches the general trend of the funds rate development hence the residual centres around zero, we can observe two corner cases:

1. The model does not capture well on the extreme volatility spikes around the era of "Fight against inflation" in early 1980s
2. Each time the funds rate drops down rapidly, the residual spikes and this is particularly visible after the dot-com bubble.

Note however, the residuals peaks/troughs do not necessarily equal to the model limit all the time. For instance, the overshoot of the funds rate estimation - hence the residual is negative - on the post-Covid era rate hike around 2022 may rather be interpreted as the FED did not hike enough in contrast to what it has done in the past considering of the inflation, GDP outgap, and the unemployment rate.

Moving onto the regime mode estimation result in Figure 2-3, we can address the style of each regime as

- Regime 0 reigns at the stable long-term rate intervals
- Regime 1 reigns at the up/downtrend of the rate regardless of the side
- Regime 2 reigns at the high volatility periods where the changes of the rate are mostly reverted back at the end

At this stage, we may better compare our result of the similar analysis conducted in [4-8]. The regime 0 has changed to the stable long-term, and the other regimes are pretty much stayed the same (their regime 1 equals our regime 2, and their regime 0 for our regime 1). This regime 0 is the new mode which significant presence in post-Lehman crisis where the FED had frozen its policy rate near zero for several years as new-normal, which never happened before hence the work in [4-8] did not need to capture as it only modelled upto year 2010.

From the calibration result in Figure 2-1, the inflation and unemployment indicators do not significantly explain much of the variance of the funds rate according to their high p-value for the t-test statistics, but only the GDP outgap does. This may be due to the fact that the lead-lag nature of the two variables where the inflation leads the funds rate and the unemployment lags, but we applied the model that only refers to those *in-phase*. One can extend the regression as VAR with lags on top of the markovian transition chain, although this would not also help for the unemployment rate's contribution as their meaningful trends develop after the funds rate's are developed.

Nonetheless, comparing the OLS parameters among the modes provide the insight of the dynamics each regime mode represents. The σ^2 variance parameter of the regime 2 is order of magnitude higher than that of regime 1 and regime 2, which means that the variance is exposed to intervals with very high volatility than usual. Also, the sensitivity to the inflation ($x1$) is significantly higher than others, which align with our observation that the regime is applied to "War on inflation" era. The sensitivity to the GDP overshoot and unemployment rate are not the cause of the rate hike as the high inflation causes the overshoot of the nominal GDP and enterprises cut jobs arrear when the significant rate hike embarks. On the other hand, the regime 0 has significantly low coefficients for all variables and constant due to the fact that it reigns on flat rate where $\Delta^{(1)} r_t$ is near zero. The regime 1's parameter values sit in between of other regimes in terms of magnitude and their signs are well-aligned with the macro-economic ground truth: the FED hikes the rate when the inflation change is positive, the actual GDP overshoots than the expected, and lowers the rate when the unemployment rate surges to alleviate the difficulties in financing for industries.

Lastly, we verify the validity of the model against the spurious regression in Figure 2-4 stating that the residual timeseries has no unit root according to the ADF and KPSS tests, and the Durbin-Watson test statistics do not show significant correlation on neither side (0 or 4) but close to 2.

Damage From the model calibration result,

1. The regime 1 being trend side agonistic is a shame that investors are keen to know which trend it goes, and the linear nature of the Markov transition as well as the OLS inference part of the switch model certainly has the potential to distinguish negatives from positives. However, enlarging the number of regime hyperparameter did not help in further experimentations.
2. Low significance of the inflation and unemployment rates, which are already discussed in the Diagnosis section.
3. As mentioned in the diagnosis section, the OLS model for the funds rate observation process do not fully capture the self-exciting volatility with the regime 2.

On the methodology design level, we also wonder if the hidden space can adopt with the human picked regime mode that represents the regime constituents of what we truly want to achieve. The modeling method in this demo does not provide the option.

Directions On the matter of "injecting hand-picked regime modes", it is possible. At the cost of undergoing manual labelling and managing them, one can inject the mode timeseries (or timeseries of probability vectors of modes) into the hidden markov chain - such as `exog_tvtp` argument for statsmodel's MarkovRegresison - and calibrate under this condition. This approach may help resolve the aforementioned "side agonistic trend regime" issue. If the model evaluates every day or shorter such that the change points to label become bigger than what humans can readily handle, one may consider conducting semi-supervised learning on the model calibration where just a part of datas to-label are marked whilst other points still subject to be filled by the hidden state space model.

Regarding the elaboration of volatility modelling, a standard solution would be to apply dedicated (G)ARCH like variance process on top of the OLS to implement the heteroskedasticity within the same regime. The I(0) stationarity of the residual also approves the application of the (G)ARCH, hence we can stack it on top of the pre-existing markov switch model.

Deployment The FED funds rate is the fore-front financial market signal that significantly transforms the expected return structure into a new market equilibrium state. For instance, a hike in rates raises the short-end government boths as well as the repo market, resulting in the yeild curve flattener. Short-end lending cost rises also transmits to the stock market lowering the real market return in general. FX rate products such as EURUSD forwards will quickly devalue the USD against EUR as the expected USD to earn increases.

As a result, the markov regime switch model's mode probability can be used as the indicator at the change point. The arrival of the change point means the market dynamics changed and the short-end product across the markets will be re-evaluated. For instance, 2025/Mar is the beginning of the trend 1 as the US government starts to execute the global tariff measure that has been causing the grim economic outlook in terms of high inflation with GDP downshoot.

One strading strategy from this model would be a trend-follower whose job is to take long/short position upon the new regime as soon as the change point is detected, to get most out of the potential return til the market returns.